Chaos: The Mathematical Beauty of Nature

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When I first offered in the summer of 1993 to give a lecture on the topic of Chaos, I thought that it would be difficult, because it was a new science, and nobody had ever heard of it outside of a few scientists and mathematicians. And then, a couple of months later, a book was published together with a film. And now, suddenly it seems that everybody knows what Chaos is, because it was an important part of that book.

The name of the book, and the film, is *Jurassic Park*.

And so those of you who have seen the film, and especially those who have read the book, will already be aware of Chaos. But I'm sure there are some of you out there who don't know about Chaos, since the science is still relatively young. And those who know it well say that it is a revolution, "one of the most important ever." But in order to understand why some people think it represents a revolution, we have to go back to the days of the last Scientific Revolution.

The Scientific Revolution took place in the 17th and 18th centuries in the physical sciences. During that revolution, man discovered many of the laws of nature that are still are considered valid today. One of the most important results of all these discoveries was that they allowed us to predict the behavior of objects in the world.

For example, Isaac Newton, the man who is famous for discovering gravity, also discovered the laws of motion. He showed that if you know the position and velocity of an object at some time (like a basketball, or even better, the planet Earth), then you could predict what would happen to that object, that is, what it will do and where it will go. This is also true in the case of our solar system. We can predict accurately exactly when the sun will rise in the morning. We can predict accurately exactly when the moon will make an eclipse of the sun.

And there are more spectacular examples of this, too: can you imagine the effect when Edmund Halley predicted *exactly* when Halley's Comet would return to earth? And that is every 86 years! And yet we are able to predict it with perfect precision, just by seeing it once and knowing how fast it is going and in what direction. The entire future, forever, of Halley's Comet, is completely determined.

This happened in many fields in science during the Scientific Revolution, not just astronomy. We discovered all of these natural laws in all sorts of fields like physics, chemistry and biology that allowed us to predict exactly what would happen in the future just by knowing enough about the present. For example, in biology, people were beginning to understand how animal populations grow. It became possible to start with a few fish in a lake and predict how many fish would be in that lake in, say,

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three months. The future even of those living animals is completely determined by the starting point of the population.

Think about it: this scientific knowledge gives us a very real power. We can "predict the future," like a witch or a magician. I'm exaggerating a little bit to try to give you a feeling for the psychological effect that the Scientific Revolution also had on the minds of people. Up until then, the only "person" who could predict the future was God Himself. And now suddenly *we* could do it, and seemingly everywhere. Suddenly all of nature was at our command — we could "play God." This was also the time when we were beginning to realize that everything in the world was made up of small particles, like molecules and atoms. And we were finding that even these small particles also obeyed those same laws of science, so that we could predict where they were going if we knew where they started.

The Scientific Revolution, the Philosophy of Science and Free Will

This new scientific power began to have an effect on our philosophical outlook at life. Philosophers began looking at all of these discoveries of natural laws, which completely determined the course of natural events; and then they began to see the consequences for our own place in the universe as human beings.

In the 18th century there was a French mathematician named Pierre Simon de Laplace who thought about this for a long time. Then, full of confidence in the almighty power of the human intellect, he claimed that if someone could tell him the position and velocity of every particle in the universe, he could predict the entire future of the universe, *forever*. Now obviously, this isn't really possible in a practical way, but think about the principle. He was saying that everything is completely determined. In particular, this implies that *Man has no free will*. We can't change the future-maybe we can learn to predict it, but we can't change it: the world is deterministic.

This is not a very nice thought, of course. Nobody likes to hear that he doesn't have a free will, that he can't do what he wants to do. This is an especially big problem for religion. Certainly the main Western religions preach that man *does* have free will.

The Discoveries of the Twentieth Century

Mr. Laplace thought that if science would just advance enough, then we would become more and more precise in our measurements over the centuries, and slowly but surely we would arrive at the point that we could predict everything through the pure application of science.

Let's stop for a minute and think about what that means. Perhaps you remember when I said in one of my lectures that up until now, it has not been *proven* that "thinking computers" (artificial intelligence) are impossible. The example I like is the invention of the helicopter by Leonardo da Vinci. There was no fundamental reason why the helicopter could not be built. Yet it took five centuries before technology was advanced enough to build the computer. Similarly, up until today, nobody has proven that it is *fundamentally* impossible to create artificial intelligence. Maybe it could take another hundred or even thousand years — enough time for technology to advance enough — but there is no proof yet that artificial intelligence cannot be achieved, if we only wait long enough for science to advance.

What about Laplace's dream of predicting the future through science, if we only wait long enough for science to become precise enough? Well, this time it is different, because the twentieth century *did* give us that proof that, in fact, it is impossible. It will *never* be possible, no matter how far science ever advances, no matter how precise our measurements ever become. There were two major discoveries in the twentieth century that gave us that proof.

The Heisenberg Uncertainty Principle

The first of these great discoveries took place during the 1930s in physics, in the discipline called quantum mechanics. That was the decade of atomic physics — in fact, it was right before they built the atomic bomb. They were doing just the kinds of things that Laplace was talking about: they were measuring the positions and the velocities of smaller and smaller particles; of atoms — indeed, of particles that were even smaller than atoms. A German physicist named Werner Heisenberg noticed something while they were doing this. He noticed that after a certain point, when things got smaller and smaller, and the measurements had to become more and more precise, that the very process of measuring interfered. Things were so delicate that just by putting your measuring instrument in there, you changed the situation so that you were no longer measuring the same thing. You were measuring something that was *changed by the very fact that you measured it.*

That may sound strange, but it isn't really. Think about people who study animals in the wild, like monkeys. We know that it is difficult to study animals in the wilderness, because they won't behave the same way if we are there, and so we use telescopes and cameras. But think about how it would be without telescopes or cameras. The only way to observe these animals would be to get close to them. But by getting close to them we change their behavior, so that we are no longer seeing their natural behavior. The result is that we cannot be really certain of what their natural behavior is. In physics, this became known as the **Heisenberg Uncertainty Principle** — it is now recognized as a fundamental law of the universe. It tells us that no matter how precisely we try to measure something, there will always be at least a small amount of uncertainty in our measurement — a small error.

Now, this was already not good news for Mr. Laplace, who hoped that someday we could make perfectly precise measurements. Mr. Heisenberg proved that it wasn't possible after all. But even this wouldn't necessarily be a disaster. After all, if we measured the speed of the earth around the sun by mistake by one meter a second too slowly, then maybe our prediction of when the sun rises would only be wrong by one second. Or maybe we would incorrectly predict when Halley's Comet would arrive by a few hours. That's not so bad, especially since we're talking about only an error of a few hours in the space of 86 years. And so even with Mr. Heisenberg's Uncertainty Principle, Mr. Laplace might still feel pretty good about his principle of determinism in the universe. There was still hope of proving that man had no free will and that everything was determined. But another great discovery of the twentieth century, in combination with Mr. Heisenberg's Uncertainty Principle, destroyed Mr. Laplace's dream of a deterministic universe forever.

Predicting the Weather

The other big discovery in the 20th century all started with one of man's biggest dreams of scientific prediction, even bigger than the dream of predicting when the sun will rise, or when Halley's Comet will return to Earth again.

It is the dream to predict the weather.

For centuries, man stood helpless in front of the weather. There were a few folkloristic sayings such as “Halo at night, sailor's delight. Halo in the morning, sailors take warning” (this was the saying that observed a halo around the moon to predict whether there would be a storm on the next day). There were almanacs that tried to predict what would happen each winter. And as a matter of fact, people did have some luck predicting what might happen the next day — or if they were really lucky, maybe even after two days.

But the big dream had always stayed out of their grasp — it was the dream of *long-range* weather forecasting — a week, even a month in the future. No one had ever succeeded, or even come close. But the 1950s were years of great optimism for long-range weather forecasting, mainly because of the rise of two technologies:

- the satellite, which could be used to observe the earth, the clouds, the oceans. It was the key technology for taking accurate measurement;

- and the computer.

There were huge and expensive weather prediction centers set up in the United States and in Europe — the famous one is in Reading, England. They used huge computers to make enormous calculations, trying to turn the prediction of the weather from a kind of black magic into a precise science.

In the early 1960s, sitting in his small laboratory at the Massachusetts Institute of Technology, was a professor named Edward Lorenz who had a very small computer program — only a couple of calculations — set up on a very small, very slow computer, trying to make a simple model on his computer of how the weather worked. People used to come by and take a look to see what the weather was doing that day in his computer program. One day there was a storm, and the next day there was clear weather. Mr. Lorenz enjoyed looking at the program and watching the “weather” unfold in front of him. Of course, it wasn't real weather, it was just a computer program simulating the weather, like a video game.

Now remember: a computer always does exactly what you tell it to do. If you start it in exactly the same way each time, then it will do exactly the same thing each time. So when Mr. Lorenz started his weather program exactly the same way two times in a row, then it would produce exactly the same weather each time.

One night he was going to get a cup of coffee, and he wanted to look a second time at what his weather program had done on that day. So he started the program again, but since he was in a hurry, he didn't put in exactly the same starting numbers as the first time. Instead, he left out the very last three decimal places in the number, which only made of difference of one thousandth. He said to himself, “Who cares about these last few numbers, they’re so small they won’t make any difference.”

When he came back from his coffee break, he couldn't believe his eyes when he looked at the program. Even though he had only changed the starting numbers by the smallest possible amount, the new weather produced by his computer program had changed so much that within only a short time it was already completely different from the original weather.

On that day, Mr. Lorenz realized, as we do now, that it is fundamentally impossible to predict the weather. (You're not surprised, are you?)

Why? Because Mr. Lorenz demonstrated with his little computer program that even the smallest change introduced at the very beginning could produce a huge, completely unpredictable effect within a relatively short time. The phenomenon that Mr. Lorenz discovered is officially known in science as *sensitive dependence on initial conditions*.

The Butterfly Effect

There is another simpler name for this phenomenon: it is called the Butterfly Effect. (they also spoke of this in the film *Jurassic Park*) The idea is that a butterfly in China flapping its wings and stirring the air could become the cause of thunderstorms in New York some time afterwards. Now, in real life the Butterfly Effect is really rather common. How many times have you heard somebody say, "If my car hadn't broken down that day, I wouldn't have met that man who helped me, who invited me to a dance, who two years later became my husband."

But in science, there was no Butterfly Effect. It was considered a universal principle for centuries that a small cause would only have a small effect. But now Mr. Lorenz showed that it wasn't true. He had given an example of a system where the smallest change could have a huge effect. And yet — and this is the amazing part — it was still a deterministic system (remember, he was using a computer program).

Thus, Mr. Lorenz showed that some systems in nature, like the weather, *even though they were deterministic*, could still behave in disorderly ways that we couldn't predict any more.

Now think about the effect of Mr. Heisenberg' Uncertainty Principle, combined with Mr. Lorenz's Butterfly Effect. Mr. Heisenberg showed that you can never be precisely sure of your initial measurements in a system. Mr. Lorenz showed that this can be a disaster for some systems, because even the smallest changes in some systems make them behave in a completely different way later on (at least after a short time).

What does this mean? This means that for these systems, it is fundamentally impossible to predict their future. Not just practically impossible, but a fundamental law of nature. It is not possible now to predict the weather, and it will not be possible a thousand years from now, even with the most powerful computers and satellites.

Of course, in today's global commercial marketplace, there are still those weather services that try to convince their customers that they can do long-term weather forecasting. In November 2001 there was a controversy in the newspapers about two commercial weather forecasting services that, in order to win customers from each other, are making larger and larger claims about the long-range accuracy of their forecasts. One of the services is the Weather Channel, and the other is **AccuWeather**.

AccuWeather recently went so far as to claim that they could make a 15-day forecast. Finally, noting the lengthening commercial forecasts, the American Meteorological Society — the largest organization of American atmospheric scientists — issued a statement essentially repudiating the idea that local weather could be accurately forecast two weeks ahead:

> “There is no scientific basis for the deterministic prediction of day-to-day weather beyond a week or two. Claims of skillful predictions of day-to-day weather changes beyond this limit have no scientific basis and are either misinformed or calculated misrepresentations of true capabilities.”

And so the dream of Mr. Laplace, to predict the future of Mankind, finally died in the twentieth century. And with this dream died his own philosophical vision of Man having no free will.

This is what Michael Crichton says in *Jurassic Park*:

> “And now chaos theory proves that unpredictability is built into our daily lives. It is as mundane as the rainstorm we cannot predict. And so the grand vision of science, hundreds of years old—the dream of total control—has died, in our century.”

The Origins of the Name “Chaos”

One of the most important characteristics of this revolution I am talking about today is that it is not taking place in just one science, like physics, but in *every* science that deals with nature in some way. This gives us a clue to what it is about.

Around the same time Mr. Lorenz was experimenting with the weather, a man named Robert May was experimenting in biology. He was looking at the way that populations of fish grow and change in a lake. Now, we know that although in theory a population of animals (or even human beings) will grow forever, but in real life sooner or later it settles down. So in a lake, when you start with a few fish, the population will grow and grow for a while, but for various reasons such as the amount of food in the lake it will reach a certain state. Now Mr. May was studying this fact closely and he was trying to see what would happen when the rate of growth of the population really became high. He found that after a couple of years, the population would no longer be steady, but rather begin to switch between two levels. One year it would be maybe 500 fish, the next year 400, then 500, then 400. If he pushed the rate of growth even harder then it would switch between four levels, like 700, 400, 500, 300, then 700 again, and so on.

So he just assumed that this would continue. Maybe the next time — the third time — he pushed the rate of growth, it would start switching between some more levels. But that's not what happened. Instead, something very strange happened. The fish population didn't switch between a few levels like before. Instead, it started changing between every possible level, with no more observable pattern. It had become completely unpredictable.

Mr. May had a colleague, a mathematician named James Yorke. Now it turns out that James Yorke knew about the work of Edward Lorenz on the weather, and the strange discoveries of Mr. Lorenz that some systems in nature, like the weather, could produce disorderly behavior even though they were deterministic.

Mr. Yorke realized that Mr. Lorenz had discovered something really new and important that wasn't just about the weather, and he noticed the similarity to what was happening with the fish populations of Mr. May.

And so James Yorke analysed this kind of strange behavior mathematically, where after only three pushes the fish population goes “crazy,” and then he wrote an article in a mathematical journal to describe the results. Mr. Yorke also had a good feeling for public relations, and he wanted to catch the eye of his readers. And so the title of this paper was:

“Period Three Implies ... Chaos”

And so a new science was born, and the name of the science was **Chaos**. This was the science of disorder in nature, but a certain kind of disorder, a disorder that nevertheless had a hidden order. It was disorderly to us, but it still behaved according to natural laws, like the ones Mr. May discovered that governed the way fish populations grow, or the one that Mr. Lorenz discovered that showed how the weather unfolds.

People like Lorenz, May and Yorke were making us realize that the disorder we saw in nature all around us wasn't just a result of our imperfect technology. The made us realize that this disorder in nature is *fundamental*, that it will *always* be with us. It made us realize that nature is supposed to be that way, and even then, there are still natural laws at work — but they are not orderly in the way they thought they were. Nature has its own kind of order that doesn't like to be described in the nice simple ways that we use. And above all, Chaos showed us that Nature isn't so predictable after all.

The Mathematics of Nature

We were also beginning to realize that the nice, logical and orderly mathematics we had developed to use to work with nature was not the right kind of mathematics, especially geometry. Up until now, we liked the so-called “Euclidean” geometry, things like nice, straight lines, circles, triangles, cones. Pisa's most famous son, Galileo Galilei, was convinced, as he once put it, that “the characters of the language in which the Book of Nature is written are triangles, circles and other mathematical figures.”

These shapes were fine when nature was orderly, but as soon as things got disorderly — “chaotic” — they weren't good enough. As one now-famous mathematician named Benoit Mandelbrot noted: “Clouds aren't circles. Mountains aren't cones. And lightning doesn't travel in a straight line.” Nature doesn't have straight lines, it's not “linear.” Most things in nature are *non-linear*. But if it wasn't the language of circles and straight lines, what was that mathematical language used to write the Book of Nature?

Mathematics in the Twentieth Century

I'd like to talk now for a couple of minutes about the problem of mathematics and its relationship to the real world. Mathematics has always been a special science, because it is pure, in the sense that it is self-contained. It can stand on its own without any relevance to the real world. In fact, that is what often happened in the past. For example, the mathematics that Einstein used to describe his Theory of Relativity was invented many years earlier, before its inventor had any idea of how it could be used to describe the real world.

Nevertheless, the twentieth century was an especially bad century for the relationship between mathematics and science. The fact is that science *does* need mathematics to help out, and in the twentieth century mathematics was becoming more and more abstract, more and more distant from anything that was happening in science. You could see this phenomenon in the attitudes of both mathematicians and scientists: Mathematicians would refuse to work on anything that wasn't pure, that had any relationship to the real, “dirty” world.

On the other side, scientists had come to think that mathematicians were just a bunch of crazy people off working in their little offices on things that nobody cared about. In this extreme world with two opposing camps — the mathematicians on one side, and the scientist on the other side — there was a mathematician named Benoit Mandelbrot. In the early 1960s (the same time that Edward Lorenz was trying to predict the weather) he was wandering over into the real world and thinking about how well mathematics described nature. And he wrote an article in a scientific journal to explain the results of some of his thinking.

The title of the article was: “How Long is the Coastline of Britain?”

Now, what a strange question to ask! It is so simple to measure the coastline of a country — just take a good measuring stick, and measure it. But Mandelbrot realized that Nature wasn't that simple. In fact, this really is a problem: Spain and Portugal have a border between them. Somebody once looked at a Spanish encyclopedia for the length of that border; and then in a Portuguese encyclopedia for the same measurement; and discovered that there was a difference of as much as *twenty percent* in their estimates. The same thing happened in Belgium and Holland, which also share a border.

So what is the problem? Suppose you look down at England from outer space, from a satellite. You can measure the coastline from the picture you see. But your measurement won't be exact because you will miss a lot of details, like the exact shape of inlets and bays. In fact, your measurement will be too short, because the indentations of the inlets make it longer. So it would be better for you to actually walk around the coastline of England. Then you could make a more exact measurement. In fact, your measurement would be longer. But even that wouldn't be good enough: you would *still* miss some details. So it would be better if, say, a mouse walked around the coastline and measured. His measurement would be even longer than yours. But this could go on forever, with more and more detail all the time. In normal geometry, the length of a line is absolutely clear-cut. But Mr. Mandelbrot showed that in Nature, even the length of a line is not clear-cut: it depends on your point of view.

He also noticed something else in Nature. The more you zoom in on the picture, the more you find the same details all over again. And Mandelbrot realized that this is the way nature is: if you break up a big stone, all the little stones have the same kind of features, irregularities; they all look like the big stone. The closer you look at a snowflake, the more detail you see. Look at how a tree grows: big branches break up into smaller branches that look the same as the big ones, and smaller ones again. An infinite amount of detail at smaller and smaller scales — and yet, all of it packed into a certain amount of space.

It's even true in biology. If you look inside the human lungs, you discover millions of little arteries all packed into this limited space, like long strings folded up next to each other. Mandelbrot started thinking about this problem of how nature packs many, many details into a certain amount of space and realized that there was also another problem. His article about the coastline of Britain had already shown how hard it was to discover the real length of a line in nature. Now he began to realize that it wasn't even clear what the *dimension* was.

Dimensions in Nature

In normal geometry a line is one-dimensional. And a piece of paper is two-dimensional. But what happens when you start to fill up the piece of paper with one long, wavy line? (Just like the arteries fill up a human lung!). The paper gets fuller and fuller. And now, suddenly the line doesn't look so one-dimensional any more, it's starting to fill up space. But it's not quite two-dimensional either, because after all, it's still only a line. In fact, it really seems like something *in between* one dimension and two dimensions. The more he thought about it the more he was convinced that this is the way that Nature really worked. And why not? It was already clear by this time that simple straight lines, circles and cones didn't exist in real Nature — and so why should it be a surprise to discover that the simple idea of one dimension and then jumping to two, then three dimensions, also didn't exist in Nature? Nature is richer than that, it offers a bigger variety of shapes and forms, and it offers a gradual shift from one dimension up to two dimensions and beyond. Mandelbrot used these new ideas to invent a new kind of mathematics for describing the real world. And in this way, Mandelbrot made a big contribution to bringing mathematics and science back together in the twentieth century.

Mandelbrot wanted to write a book about his discoveries, but he needed a name for this new kind of geometry he had discovered to talk about nature. One day his son came home from school, and brought his dictionary for Latin class. Mandelbrot looked through this Latin dictionary for some ideas and he found the word *fractus*, which comes from the word *frangere*, which means "to break." He liked that idea, because it corresponded to his vision of Nature and the way it breaks up into smaller and smaller "fractured" pieces, and the idea of dimensions that are not whole numbers like one or two, but fractions. And so he invented a new word from this, and the title of his book became: *The Fractal Geometry of Nature*. So now we have it. Galileo was wrong: the Book of Nature was not written in the language of circles and lines—it was written in the language of *fractals*.

Fractals

Unlike the mathematics for Einstein's Theory of Relativity, the mathematics of fractals didn't have to wait for many years for a science where it could be applied. Fractals turned out to be the mathematics of chaos. Chaos was the science of real nature in all of its rich and varied disorder, and fractals were its geometry.

Mandelbrot also made a big contribution in bringing mathematics together with another invention of the twentieth century: the computer. The computer may seem very mathematical to you, but in fact it's not. It is stupid, it calculates only exactly what you tell it to calculate, and most pure mathematicians hate the computer. They are proud of their ability to construct elegant theories without that big stupid machine, and in fact they basically mistrust it. After all, mathematics has been existence for centuries, and all of those beautiful theories, some of the greatest feats of man's imagination, were developed without the computer.

But computers turned out to be perfect for exploring Mandelbrot's new fractal geometry. Fractals consist of many, many simple structures that repeat themselves in more and more detail — just as in Nature. And there's one thing about computers you cannot deny: maybe they can only do simple calculations, but they can do a lot of them *very quickly*. Furthermore, this new fractal mathematics has the problem that with so many of these simple, detailed calculations, it is no longer possible to draw a picture with a pencil and paper. It's just too much for a human being. But thanks to the rapid advance of computer graphics, computers can handle this very well, because they made it possible to draw pictures that consisted of millions of smaller and smaller details. You can see a simple example of this in the book *Jurassic Park*. Each chapter begins with a fractal that is calculated in more and more detail, starting with its simplest form in the beginning of the first chapter, all the way up to a very elaborate form in the last chapter. (That fractal is also called the "dragon fractal" because it looks a bit like a dragon).

This marks the beginning of a new age of using the computer, and especially computer graphics, to help in mathematical research.

Fractal Landscapes

One persuasive indication that fractals are the geometry of nature is that the pictures created by fractals such as the Mandelbrot set look real — they look *right*. Fractals are being used in the movie industry to create imaginary mountain landscapes for films. For example, one of the Star Trek movies (the second one, I believe) used fractals to show the birth of a new planet including its mountains and rivers.

The Beauty of Fractals

But there is one thing about computer-generated fractal pictures that I believe is the ultimate proof that they really represent nature. They are *beautiful*. Not in an abstract way, like the skyline of Manhattan, with its straight-lined skyscrapers, but beautiful in the way that nature is beautiful to us.

To illustrate what I mean, let’s look at a series of images created by a number of fractal artists — these are artists who make use of fractal-generating programs to create visual art.

While you're looking at these images, consider the following words spoken by a scientist:

> "Why is it that the [shape] of a storm-bent tree with no leaves against a sky in winter is perceived by us to be beautiful, but the shape of some normal building is not, in spite of all efforts by the architect? [Because] our feeling for beauty is inspired by the harmonious arrangement of order and disorder as it occurs in natural objects — in clouds, trees, mountain ranges, or snow crystals."

In fact, the fractals are so beautiful that there was a traveling art exhibition a few years ago that went around the world, displaying fractals. There have also been several collections of fractals in books.

The Many Applications of Chaos Theory

Scientists began to discover chaos and fractals in all kinds of areas: human physiology, plants, the flow of rivers; and even in areas like economics. Mandelbrot discovered that the price of cotton over several years on the stock market obeyed his fractal mathematics!

Chaos in Business Organization

I have a colleague at the National Research Council of Canada, whose name is Hakan Erdogmus. He has pointed out to me that in the 1990s, many business managers became fascinated with the application of Chaos theory to business. To illustrate this, he cited a book by Ralph Stacey, called *Managing the Unknowable: Strategic Boundaries Between Order and Chaos in Organizations*. Here is an excerpt from that book:

> A business climate is basically a non-linear system, and often it's those small, unpredictable changes that escalate and shape the organization. The result is a system of bounded instability, and the successful organization oscillates within the many end points of this instability, for no organization can sustain a heightened level of stability for long periods of time in an uncertain climate without failing in the end. Aiming for equilibrium actually destroys innovation.
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> The best that management can do is to create the conditions for unbounded instability. For it to happen, contention, political interaction, and group learning (in which teams of individuals follow an agenda, interact, learn, and modify their actions and beliefs) are necessary. Issues need to be raised, discussed, and contended among staff continuously. This process leads to unexpected actions, and ultimately innovation, some of which end up creating a lot of value.

Fractal Music

The piece of music you heard in the beginning of this lecture was fractal music. Here is what the composer said about its creation:

> This piece was created and copyrighted © 1996 by Forrest Fang using Robert Greenhouse's program, *The Well-Tempered Fractal*, which is available from the Fractal Music Project website. The program generates MIDI data from different types of fractals, according to certain parameters that the user sets. The one here

> is based on the Duffing attractor: Several different runs of the program were "assembled" on a commercial sequencer, and their data were slightly randomized by Mr. Fang according to whim. He then transposed the different voices into octaves that seemed to aurally complement each other. Despite all of the technical aspects involved in creating the data, the piece was generated intuitively.

Chaos and Ethics

The principles of Chaos have even been applied to the study of ethics, to the moral and religious ways in that we view life. A colleague of mine, Massimo d'Alessandro, is a teacher of Bio-Dancing, which uses movement as well as philosophical reflections to give students new perspectives on life. This is what he wrote to me about his application of the principles of Chaos in his work:

> The idea I would like to propose is that [chaos gives us] a principle of *order hidden inside of turbulence*. This means that even the apparent and incomprehensible chaos that happens in our lives, in reality can hide a secret trajectory that is in fact well ordered.
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> Many cultural, and especially, religious and ideological mandates, strive directly for perfect order, purity, justice, coherence, integrity, but they clash with our interior nature, which isn't like that at all. They make us feel like "sinners", incapable of living according to this ideal religious or moral or social order. But in reality, we can view this perfect order and purity as something that we can arrive at only by in reality generating turbulences and apparently disordered actions.
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> In [summary]: *harmony can only be achieved by passing first through chaos*.

Chaos and Free Will

I'd like to finish the lecture by coming back to what we were discussing at the beginning about Mr. Laplace and his desire to prove that man has no Free Will - that the world is completely determined and predictable. Now we know that Nature is not so simple, but that it is full of processes that are "chaotic" and unpredictable. Now some scientists have started to ask the following question: "What if the laws of Chaos govern our minds?" If that is true, then small fluctuations in the processes in our brains are amplified (through the Butterfly Effect) into thoughts, in many kinds of unpredictable ways, so that many of these become what we call "creative" thoughts. They are *deterministic*, but they are *unpredictable*, just like other chaotic phenomena in the real world.

In other words, scientists are now looking to Chaos as a way of making it possible to think about a world that is deterministic, as Mr. Laplace thought, but in which it is also possible for man to have Free Will. And so you see, with or without Chaos, and any other revolution in science that is still to come, the great philosophical questions will continue to be asked, and maybe will never be resolved.

References

There are many fractal-generating programs available. The `fractint` program (the one used for the demonstrations in this lecture) is the oldest and still in many ways the best of the fractal-generating programs. And best of all, it is free. Look at the website `www.fractint.org.`

The fractal art was taken from the submissions to the Usenet Newsgroup `alt.binaries.fractal-art`.

The fundamental book to read, a treatment of Chaos for the layman, is *Chaos: Making a New Science*, by James Gleick, Penguin Books, 1988, ISBN 0 14 00.9250 1. It has been translated into Italian: *Caos: La nascita di una nuova scienza*, James Gleick, RCS Rizzoli Libri S.p.A., Milano, ISBN 88-17-85248-1, maggio 1989.

Here is the reference for the book on management and Chaos mentioned in the lecture: *Managing the Unknowable : Strategic Boundaries Between Order and Chaos in Organizations* (Jossey-Bass Management Series), by Ralph D. Stacey.